Important Formulae for Engineering Mathematics

LAWS OF INDICES

1) $a \times a \times a \times a \times$ ..... n times $= a^n$

2) $a^m \times a^n = a^{m+n}$

3) $a^m \div a^n = a^{m-n}$

4) $(a^m)^n = a^{mn}$

5) $(ab)^n = a^n \times b^n$

6) $(a \div b)^n = a^n \div b^n$

7) $a^0 = 1$

8) $a^{-m} = \frac{1}{a^m}$

IDENTITIES

1) $(a \pm b)^2 = a^2 \pm 2ab + b^2$

2) $(a + b)(a - b) = a^2 - b^2$

3) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

$= a^3 \pm b^3 \pm 3ab(a \pm b)$

4) $(a^3 \pm b^3) = (a \pm b)(a^2 \mp ab + b^2)$

$= (a \pm b)^2 \mp 3ab(a \pm b)$

5) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

6) $a^3 + b^3 + c^3 - 3abc = (a + b + c)$

$(a^2 + b^2 + c^2 - ab - bc - ca)$

VOLUME AND SURFACE AREA

Sr.No. & Name	Curved Surface Area	Total Surface Area	Volume
1. CUBOID	-	$2(lb + bh + lh)$	$l \times b \times h$
2. CUBE	-	$6l^2$	l^3
3. CYLINDER	$2\pi rh$	$2\pi r(h + r)$	$\pi r^2 h$
4. CONE	πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$
5. SPHERE	-	$4\pi r^2$	$\frac{4}{3}\pi r^3$

LOGARITHM

1. If $a^x = N$ then $\log_a N = x$
2. $\log_a m + \log_a n = \log_a (m\,n)$
3. $\log_a m - \log_a n = \log_a \left[\frac{m}{n}\right]$
4. $\log_a (m)^n = n \log_a m$
5. $\log_a b = \frac{\log_e b}{\log_e a}$ change of base formula
6. $\log_a b = \frac{1}{\log_b a}$
15. $\log_a e^x = x$
7. $\log_e 1 = 0$, $\log_e 0 = -\infty$, $\log_e e = 1$, $\log \infty = \infty$,

$e^{\infty} = \infty$, $e^{-\infty} = 0$

RELATION BETWEEN DEGREES & RADIANS :

1 radian $(1^C) = \frac{180}{\pi}$ degree $= (180 \times \frac{7}{22})^\circ = 57^\circ\ 16'$

1 degree $(1^0) = \frac{\pi}{180}$ radian $= (\frac{22}{7 \times 180})^C = 0.01746^C$

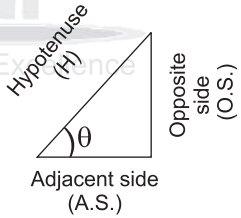
TRIGONOMETRY :

TRIGONOMETRIC RATIOS :

$\sin\theta = \frac{O.S.}{H}$ $\quad \operatorname{cosec}\theta = \frac{H}{O.S.}$

$\cos\theta = \frac{A.S.}{H}$ $\quad \sec\theta = \frac{H}{A.S.}$

$\tan\theta = \frac{O.S.}{A.S.}$ $\quad \cot\theta = \frac{A.S.}{O.S.}$



RELATIONS :

i) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad$ ii) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

iii) $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ iv) $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

v) $\cot\theta = \frac{1}{\tan\theta}$

IDENTITIES :

i) $\sin^2\theta + \cos^2\theta = 1$ $\quad$ ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

ADDITION FORMULAE :

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

FACTORIZATION FORMULAE

$\sin C + \sin D = 2 \sin\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

$\sin C - \sin D = 2 \cos\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right)$

$\cos C + \cos D = 2 \cos\left(\dfrac{C+D}{2}\right) \cos\left(\dfrac{C-D}{2}\right)$

$\cos C - \cos D = -2 \sin\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{C-D}{2}\right)$

$= 2 \sin\left(\dfrac{C+D}{2}\right) \sin\left(\dfrac{D-C}{2}\right)$

S + S = 2 SC
S – S = 2 CS
C + C = 2 CC
C – C = – 2 SS

DE FACTORIZATION FORMULAE :

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

2 SC = S + S
2 CS = S – S
2 CC = C + C
2 SS = C – C

MULTIPLE ANGLE FORMULAE

1. Double Angle Formulae :

$\sin 2A = 2 \sin A \cos A$

$= \dfrac{2 \tan A}{1 + \tan^2 A}$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2 \cos^2 A - 1$

$= 1 - 2 \sin^2 A$

$= \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

2. Triple Angle Formulae

$\sin 3A = 3 \sin A - 4 \sin^3 A,$

$\cos 3A = 4 \cos^3 A - 3 \cos A,$

$\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

Remark :

1) $1 - \cos 2\theta = 2 \sin^2 \theta$

$1 + \cos 2\theta = 2 \cos^2 \theta$

2) $1 + \sin 2\theta = [\sin \theta + \cos \theta]^2$

$1 - \sin 2\theta = [\sin \theta - \cos \theta]^2$

* $\frac{1+\tan\theta}{1-\tan\theta} = \tan\left[\frac{\pi}{4}+\theta\right]$

$\frac{1-\tan\theta}{1+\tan\theta} = \tan\left[\frac{\pi}{4}-\theta\right]$

$\frac{\tan\theta-1}{\tan\theta+1} = \tan\left[\theta-\frac{\pi}{4}\right]$

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$
$\cot(-\theta) = -\cot\theta$
$\sec(-\theta) = \sec\theta$
$\text{cosec}(-\theta) = -\text{cosec}\,\theta$

* $\sin 0 = 0, \sin\pi = 0, \sin n\pi = 0, \sin 2n\pi = 0, \sin \pi/2 = 1$
$\cos 0 = 1, \cos\pi = -1, \cos n\pi = (-1)^n \cos 2n\pi = 1, , \cos \pi/2 = 0$

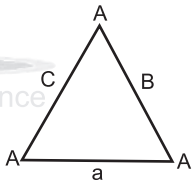
INVERSE TRIGONOMETRIC FUNCTIONS

1. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
   $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$
   $\cot^{-1}x + \tan^{-1}x = \frac{\pi}{2}$
2. $\text{cosec}^{-1}x = \sin^{-1}\frac{1}{x}$
   $\sec^{-1}x = \cos^{-1}\frac{1}{x}$
   $\cot^{-1}x + \tan^{-1}\frac{1}{x}$
3. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$\sin^{-1}(-x) = \sin^{-1}x$
$\cos^{-1}(-x) = \pi - \cos^{-1}x$
$\tan^{-1}(-x) = -\tan^{-1}x$
$\cot^{-1}(-x) = \pi - \cot^{-1}x$
$\sec^{-1}(-x) = \pi - \sec^{-1}x$
$\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x$

SINE FORMULAE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



COSINE FORMULAE

$\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos B = \frac{c^2+a^2-b^2}{2ca}$

$\cos C = \frac{a^2+b^2-c^2}{2ab}$

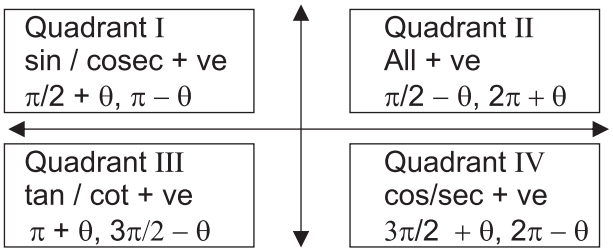
AREA OF TRIANGLE

Area $= \frac{1}{2}$ bc sin A
$= \frac{1}{2}$ ca sin B
$= \frac{1}{2}$ ab sin C

PROJECTION RULE

a = b cos C + c cos B
b = c cos A + a cos C
c = a cos B + b cos A

Trigonometric Ratios of Different Quadrants



1. If angle is along X-axis keep the ratio and think of sign. e.g. $\sin(\pi - \theta) = \sin\theta$
2. If angle is along Y-axis change the ratio and think of sign. e.g. $\cos(\pi/2 + \theta) = -\sin\theta$

(change is from sin to cos, tan to cot, sec to cosec)

LIMITS

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1,$
2. $\lim_{x \to 0} \frac{\tan x}{x} = 1,$
3. $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1,$
4. $\lim_{x \to 0} \frac{a^x - 1}{x} = \log a,$
5. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
6. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
7. $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$

DERIVATIVE

Function	Derivative
x^n	nx^{n-1}
$1/x$	$-1/x^2$
e^x	e^x
a^x	$a^x \log a$
$\log x$	$1/x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\operatorname{cosec}^2 x$

Function	Derivative
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$
$u \times v$	$u\frac{dv}{dx} - v\frac{du}{dx}$
$\frac{u}{v}$	$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

INTEGRATION

Function	Integration
x^n	$\frac{x^{n+1}}{n+1} + C$
$1/x$	$\log x + c$
e^x	$e^x + c$
a^x	$\frac{a^x}{\log a} + c$
sinx	–cosx + c
cosx	sinx + c
tanx	log |secx| + c
cotx	log |sin x| + c
secx	log |secx + tanx| + c
cosec x	log|cosecx – cotx| + c
sec^2x	tanx + c
$cosec^2x$	–cotx + c
secx tanx	secx + c
cosecx cotx	–cosecx + c
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\frac{x}{a} + c$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\log (x + \sqrt{x^2 - a^2}) + c$
$\frac{1}{\sqrt{x^2 + a^2}}$	$\log (x + \sqrt{x^2 + a^2}) + c$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\log\left\|\frac{a + x}{a - x}\right\| + c$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\log\left\|\frac{x - a}{x + a}\right\| + c$
$\sqrt{a^2 - x^2}$	$\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$
$\sqrt{x^2 - a^2}$	$\frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log (x + \sqrt{x^2 - a^2}) + c$
$\sqrt{x^2 + a^2}$	$\frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log (x + \sqrt{x^2 + a^2}) + c$

1. $\int e^x [f(x) + f'(x)]dx = e^x f(x)$

2. $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

3. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)}$

4. $\int [f(x)]^n f'(x)\, dx = \frac{f(x)^{n+1}}{n+1}$

5. $\int e^{f(x)} f'(x)\, dx = e^{f(x)}$

6. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a\cos bx + b\sin bx)$

7. $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2} (a\sin bx - b\cos bx)$

8. $\int u\, v\, dx = u \int v\, dx - \int \left[\frac{du}{dx} \int v\, dx\right] dx$

9. $\int u\, v\, dx = u\, v_1 - u\, v_2 + u''v_3 - \ldots.$

DEFINITE INTEGRAL :

1. $\int_a^b f(x)\, dx = \int_a^b f(y)\, dy = \int_a^b f(t)\, dt$

2. $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

3. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

4. $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

5. $\int_0^a f(x)\, dx = \int_0^a f(a - x)\, dx$

6. $\int_0^{2a} f(x)\, dx = \int_0^a [f(x) + f(2a - x)]\, dx$

7. $\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx & \text{If } f(x) \text{ is even} \\ 0 & \text{If } f(x) \text{ is odd} \end{cases}$

Imaginary Unit : $i = \sqrt{-1}$

$i^2 = -1,\ i^3 = -i,\ i^4 = 1,\ i^5 = 1,\ 1/i = -i$

Value of any power of i lies between 1, –1, i and –i

Complex Number :

1. Cartesian Form : $z = x + iy$
2. Polar Form : $z = r(\cos\theta + i\sin\theta)$
3. Exponential Form : $z = re^{i\theta}$

Remarks : 1. Modulus : $|z| = r = \sqrt{x^2 + y^2}$

2. Amplitude / Argument : $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

3. Euler's Formulae :

$e^{i\theta} = \cos\theta + i\sin\theta,\ e^{-i\theta} = \cos\theta - i\sin\theta$

$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$, $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

4. De Moivre's Theorem :

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Hyperbolic Functions :

$\cosh x = \frac{e^x + e^{-x}}{2}$, $\sinh x = \frac{e^x - e^{-x}}{2}$

$\sinh(-x) = -\sinh x \Rightarrow$ odd function

$\cosh(-x) = \cosh x \Rightarrow$ even function

$\cosh^2 x - \sinh^2 x = 1$ $\quad \sinh^{-1} x = \log[x + \sqrt{x^2 + 1}]$

$\tanh^2 x + \operatorname{sech}^2 x = 1$ $\quad \cosh^{-1} x = \log[x + \sqrt{x^2 - 1}]$

$\coth^2 x - \operatorname{cos\,ech}^2 x = 1$ $\quad \tanh^{-1} x = \frac{1}{2}\log\left[\frac{1+x}{1-x}\right]$

Logarithm of Complex Number :

$\log(x + iy) = \log\sqrt{x^2 + y^2} + i\left(2n\pi + \tan^{-1}\frac{y}{x}\right)$

$\log(x + iy) = \log\sqrt{x^2 + y^2} + i\tan^{-1}\frac{y}{x}$

Quadratic Equation :

$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac = 0 \rightarrow$ Real and equal roots

$b^2 - 4ac > 0 \rightarrow$ Real and unequal roots

$b^2 - 4ac < 0 \rightarrow$ Roots are complex conjugates

STANDARD EXPANSIONS :

1. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$
2. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots.$
3. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots.$
4. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots..$
5. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots..$
6. $\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \ldots.$
7. $\log (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots.$
8. $\log (1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \ldots.$
9. $\frac{1}{1 + x} = 1 - x + x^2 - x^3 + \ldots.$
10. $\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \ldots.$
11. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots..$

Factorial : $n! = 1 \times 2 \times 3 \times \ldots \times n$

$0! = 1$

Permutation: $^nP_r = \frac{n!}{(n - r)!}$

Combination : $^nC_r = \frac{n!}{(n - r)!\, r!}$

$^nC_n = 1,\ ^nC_0 = 1,\ ^nC_r = {}^nC_{n-r},\ ^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Probability : $p(A) = \frac{n(A)}{n(s)}$ $\quad 0 \leq P \leq 1$

Arithmetic progression :

a, (a + d), (a + 2d), (a + 3d) ....., {a + (n - 1)d}

n^{th} term : $T_n = a + (n - 1)\, d$, a = 1st term,

d = common difference

Sum of n terms : $S_n = n/2\ \{2a + (n - 1)\, d\}$

Geometric progression :

a, ar, ar^2, ..., ar^{n-1} n^{th} term of a G.P. $T_n = ar^{n-1}$

Sum of n terms of a G.P.

$S_n = \frac{a(1-r^n)}{1-r}$, if $r < 1$ $S_n = \frac{a(r^n-1)}{r-1}$, if $r > 1, r \neq 1$

Sum to infinity of a G.P. $S_\infty = \frac{a}{1-r}$

Co-ordinate Geometry :

1. Distance Formulae; $A(x_1, y_1)$, $B(x_2, y_2)$

$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

2. Section Formulae :

Internal Division $R \equiv \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

External Division $R \equiv \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

3. Midpoint : $M \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

4. Centroid : $\equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Line :

Slope (m) = $\tan\theta = \frac{y_2-y_1}{x_2-x_1} = \frac{-\text{coeff. of } x}{\text{coeff. of } y}$

Equations of Line :

1. $(y - y_1) = m(x - x_1)$, 2. $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

3. $y = mx + c$, 4. $ax + by + c = 0$

5. $\frac{x}{a} + \frac{y}{b} = 1$ (a and b are x and y intercepts)

*$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

*Parallel lines : $m_1 = m_2$

Perpendicular line : $m_1 m_2 = -1$

*Length of perpendicular = $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Circle :

1. $x^2 + y^2 = r^2$

2. $(x - h)^2 + (y - k)^2 = r^2$, $(h, k) \equiv$ centre

Parabola :

$y^2 = 4ax$, $y^2 = -4ax$, $x^2 = 4ay$, $x^2 = -4ay$

Ellipse : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, **Hyperbola :** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vector :

1. Dot (Scalar/Inner) product :

$\bar{a} \cdot \bar{b} = |\bar{a}|\,|\bar{b}| \cos\theta = ab\cos\theta \ldots 0 \le \theta \le \pi$

i) Angle between $\bar{a}$ and $\bar{b}$ is $\cos\theta = \frac{\bar{a} \cdot \bar{b}}{ab}$

ii) $\bar{a} \cdot \bar{b} > 0$, θ is acute, $\bar{a} \cdot \bar{b} < 0$, θ is obtuse.

iii) $\bar{a} \perp \bar{b}$ i.e. $\theta = 90$, $\cos 90 = 0$ $\therefore \bar{a} \cdot \bar{b} = 0$

$\bar{i} \cdot \bar{j} = \bar{j} \cdot \bar{k} = \bar{k} \cdot \bar{i} = 0$

iv) $\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$ commutative

v) $\bar{a}$ collinear with $\bar{b}$ i.e. $\theta = 0$, $\cos 0 = 1$

$\therefore \bar{a} \cdot \bar{b} = ab$

$\bar{i} \cdot \bar{i} = \bar{j} \cdot \bar{j} = \bar{k} \cdot \bar{k} = 1, \bar{a} \cdot \bar{a} = a^2$

vi) If $\bar{r}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ $\bar{r}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

then $\bar{r}_1 \cdot \bar{r}_2 = a_1a_2 + b_1b_2 + c_1c_2$

2. Cross Product :

$\bar{a} \times \bar{b} = |\bar{a}|\,|\bar{b}| \sin\theta\,\hat{n}$, $|\bar{a} \times \bar{b}| = |\bar{a}|\,|\bar{b}| \sin\theta$

i) Vector product is not commutative

$\bar{a} \times \bar{b} \neq \bar{b} \times \bar{a}$ $\therefore \bar{a} \times \bar{b} = -(\bar{b} \times \bar{a})$

ii) $\bar{a} \times \bar{a} = \bar{0}$, $\bar{a}$, $\bar{b}$ are collinear iff $\bar{a} \times \bar{b} = \bar{0}$

$\hat{i} \times \hat{i} = \bar{0}$ $\hat{j} \times \hat{j} = \bar{0}$ $\hat{k} \times \hat{k} = \bar{0}$

$\hat{i} \times \hat{j} = \hat{k}$ $\hat{j} \times \hat{k} = \hat{i}$ $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$ $\hat{k} \times \hat{j} = -\hat{i}$ $\hat{i} \times \hat{k} = -\hat{j}$

iii) If $\bar{r}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ $\bar{r}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$, the

$$\bar{r}_1 \times \bar{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

3. Scalar Triple Product :

$$(\bar{a} \times \bar{b}) \cdot \bar{c} = [\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

4. Vector Triple Product :

$$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\,\bar{b} - (\bar{a} \cdot \bar{b})\,\bar{c}$$

5. Vector Quadruple Product :

$$(\bar{a} \times \bar{b}) \cdot (\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{d} \\ \bar{b}\cdot\bar{c} & \bar{b}\cdot\bar{d} \end{vmatrix}$$

$$(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{b}\ \bar{d}]\,\bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\,\bar{d}$$

PHYSICAL QUANTITIES :

Sr. No.	Physical Quantity	Formula	Unit
1.	Velocity (v)	$v = \frac{\text{displacement}}{\text{time}} = \frac{ds}{dt}$	m/s
2.	Acceleration (a)	$a = \frac{\text{Change in velocity}}{\text{time}} = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	m/s^2
3.	Momentum (P)	P = m v	kg m/s
4.	Force (F)	F = m a	N or Kg m/s^2
5.	Impulse (I)	I = Force × Change in time	N s
6.	Work (W)	W = F.S.	N m or J
7.	Power (P)	$P = \frac{W}{t} = \frac{F.S}{t} = F v$	J/s or W
8.	Pressure (P)	$P = \frac{F}{A}$	N/m^2
9.	Density (ρ)	$\rho = \frac{m}{v}$	kg/m^3
10.	Kinetic Energy (K.E.)	$K.E. = \frac{1}{2} m v^2$	J
11.	Potential Energy (P.E.)	P.E. = m g h	J

Equations of Motion :

$v = u + at$

$s = ut + \frac{1}{2} a t^2$

$v^2 = u^2 + 2 a s$

Curvilinear Motion :

Direction	Velocity Comp.	Acceleration Comp
x	$v_x = \frac{dx}{dt}$	$a_x = \frac{dv_x}{dt}$
y	$v_y = \frac{dy}{dt}$	$a_y = \frac{dv_y}{dt}$
tangential (t)	$v_t = v$	$a_t = \frac{dv}{dt}$
normal (n)	$v_n = 0$	$a_n = \frac{v^2}{\rho}$
radial (r)	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
transverse (θ)	$v_\theta = r\dot{\theta}$	$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

RECTILINIAR MOTION :

Motion Diagram:

I] v-t diagram :

- slope = Acceleration $= \frac{dv}{dt}$
- Area under v-t curve = change in position $= (x_2 - x_1)$
- displacement = Algebraic sum of area of v - t diagram
- distance = sum of area (all +ve) of v - t diagram
- for vel. to be max. $\frac{dv}{dt} = a = 0$

II] a-t diagram :

- slope = Jerk = $\frac{da}{dt}$
- Area = change in velocity $= (V_2 - V_1)$
- To find position use moment equation $x_t = x_0 + v_0 t + M_t$

III] x-t diagram :

- slope = velocity = $\frac{dx}{dt}$
- for x to be max. $\frac{dx}{dt} = v = 0$

IV] v-x diagram :

- Accleration = velocity × slope of v–x diagram

v] a-x diagram : Area = $\frac{v_2^2 - v_1^2}{2}$

PROJECTILE MOTION :

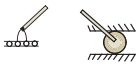
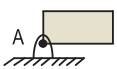
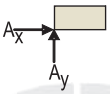
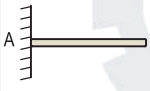
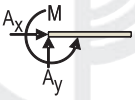
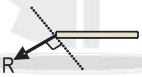
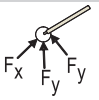
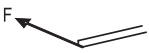
Sr. No.	Term	Horizontal Plane	Inclined Plane
1.	Time of flight (t)	$t = \frac{2u \sin \alpha}{g}$	$t = \frac{2u \sin \alpha}{g \cos \theta}$
2.	Range (R)	$R = \frac{u^2 \sin 2\alpha}{g}$	$R = \frac{2u^2 \sin \alpha}{g \cos^2 \theta} \cos(\alpha + \theta)$
3.	Angle of projection for max. Range	$\alpha = 45^0 = \frac{\pi}{4}$	$\alpha = \frac{\pi}{4} - \frac{\theta}{2}$
4.	Max. Range	$R_{max} = \frac{u^2}{g}$	$R_{max} = \frac{u^2}{g(1+\sin\theta)}$
5.	Max. Height	$H = \frac{u^2 \sin^2\alpha}{2g}$	$H = \frac{u^2 \sin^2\alpha}{2g \cos \theta}$
6.	Equation of Trajectory	$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2\alpha}$	-

WORK ENERGY PRINCIPLE

work = U = F × S, Scalar quantity, Unit - N m (J)

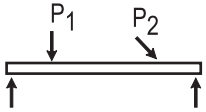
Sr. No.	Type of Force	Work Done
1.	**External force (constant)**	$= F \times S$
2.	**Variable force**	$= \int_{s_1}^{s_2} F \, ds$
3.	**Frictional force** (i) Horizontal plane	$= -\mu_k \, mg \, s$
	(ii) Inclined plane	$= -\mu_k \, mg \cos\theta \, S$
4.	**Gravity force** (i) Horizontal plane	$= 0$
	(ii) Inclined plane	$= \pm mg \, S \sin\theta$
5.	**Spring force** (i) not connected to particle	$= \frac{1}{2} k [x_1^2 - x_2^2]$
	(ii) connected to particle	$= \frac{1}{2} k [(l_1 - l_0)^2 - (l_2 - l_0)^2]$

TYPES OF SUPPORTS & CORRESPONDING REACTIONS :

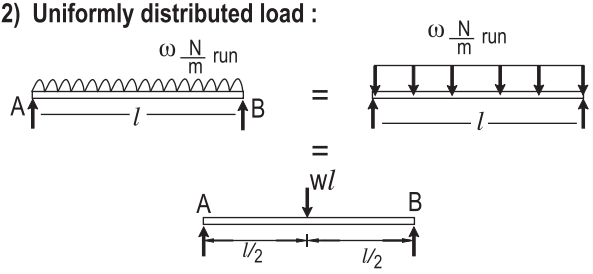
Sr. No.	Support/ Connection	Sketch	Reaction	Specification	No. of unknowns
1.	Rollers			Known reaction which is $\perp^{er}$ to plane of roller	One
2.	Smooth surface			Reaction is $\perp^{er}$ to the surface	One
3.	Rough surface			Two reaction components with unknown directions	Two
4.	Smooth pin or Hinge			Two reaction components with unknown directions	Two
5.	Flexible cord, rope or cable of negligible weight			One axial force acting away from body (Tension)	One
6.	Fixed support			Two reaction components and one moment with all components unknown in directions.	Three
7.	A smooth pin in a slot			Reaction with known line of action which is always $\perp^{er}$ to slot in which pin is sliding.	One
8.	A sliding collar			Reaction is perpendicular to the rod along which collar is sliding without friction	One
9.	Ball and socket joint			Three reaction components in unknown directions.	Three
10.	A short link			Force with known line of action	One

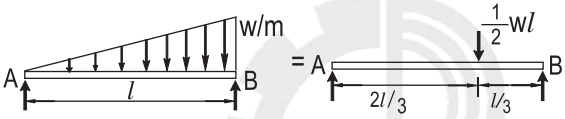
TYPES OF LOADS ON THE BEAM

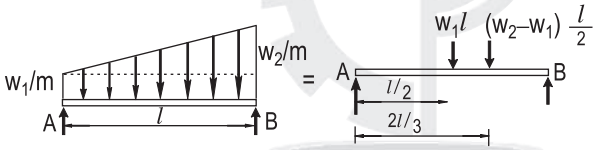
1) Point / concentrated Load :

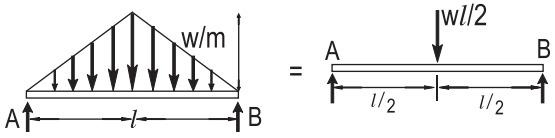


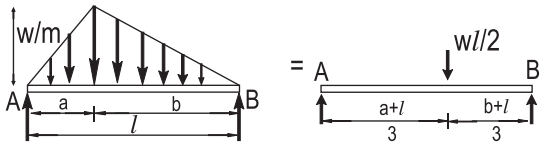
2) Uniformly distributed load :



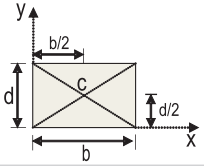
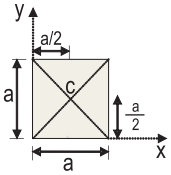
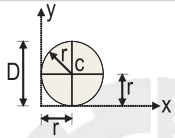
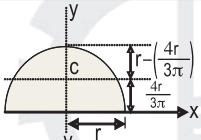
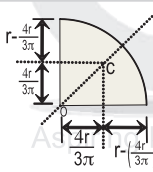
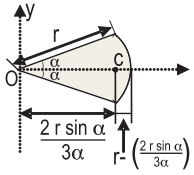
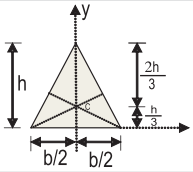
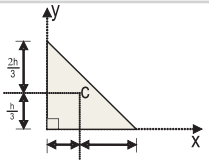




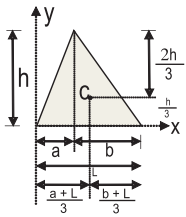
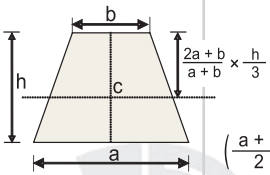
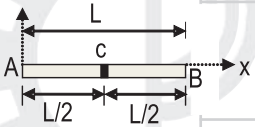
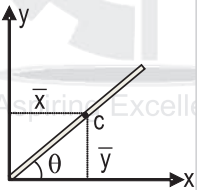
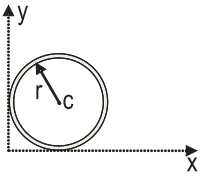
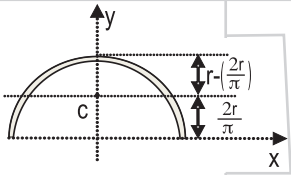




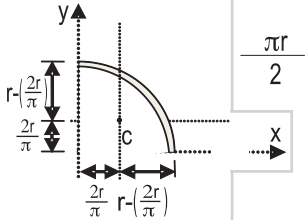
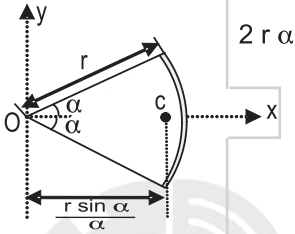
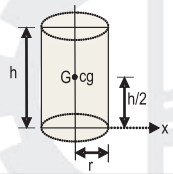
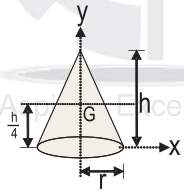
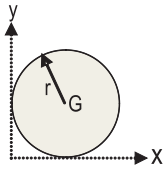
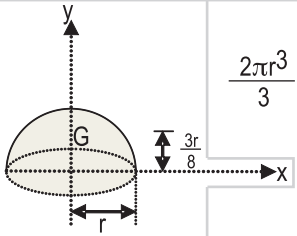
CENTROID :

Sr. No.	Name	Shape	Area	$\overline{X}$	$\overline{Y}$
1	Rectangle		b × d	b/2	d/2
2	Square		a^2 OR $\frac{(\text{Diagonal})^2}{2}$	$\frac{a}{2}$	$\frac{a}{2}$
3	Circle		πr^2 OR $\frac{\pi}{4}D^2$	r	r
4	Semi circle		$\frac{\pi r^2}{2}$	0	$\left(\frac{4r}{3\pi}\right)$
5	Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
6	Sector of a circle		Note $\alpha\, r^2$ (α in radian) α is semiangle	$\frac{2r\sin\alpha}{3\alpha}$ (α in degree) / (α in radian)	0
7 (a)	Triangle Symmetrical triangle		$\frac{1}{2}\times b\times h$	0	h/3
(b)	Right angled triangle		$\frac{1}{2}\times b\times h$	b/3	h/3

CENTROID :

Sr.No.	Name	Shape	Area	$\overline{X}$	$\overline{Y}$
(c)	Unsymmetrical triangle		$\frac{1}{2} \times L \times h$	$\frac{a+L}{3}$	$\frac{h}{3}$
8.	Trapezoid		$\left(\frac{a+b}{2}\right)h$	-	-
Line segments :					
9.	A straight line		L	L/2	0
10.	An inclined line		L	$\frac{L}{2}\cos\theta$	$\frac{L}{2}\sin\theta$
11.	Circular Arc		$2\pi r$	r	r
12.	Semi circular Arc		πr	0	$\frac{2r}{\pi}$

CENTROID :

Sr.No.	Name	Shape	Area	$\overline{X}$	$\overline{Y}$
13	Quarter circular Arc		$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
14	An arc of a circle		$2 r \alpha$	$\frac{r \sin \alpha}{\alpha}$	0
15	A solid cylinder		$\pi r^2 h$	0	h/2
16	A solid right circular cone		$\frac{1}{3}\pi r^2 h$	0	$\frac{h}{4}$
17	Sphere		$\frac{4}{3}\pi r^3$	r	r
18	Hemi-Sphere		$\frac{2\pi r^3}{3}$	0	$\frac{3r}{8}$